Performance Based Learning and Assessment Task

At the Gas Pump

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

This task is to provide students with the opportunity to analyze a real world situation and create a mathematical model. Students will use functions and make predictions using their algebraic equations.

II. UNIT AUTHOR:

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III. COURSE:

Algebra I, Algebra II, AFDA

IV. CONTENT STRAND:

- Algebra and Functions

V. OBJECTIVES:

The learner will be able to find an equation that models the data, graph the function, and use the information to predict future gasoline and diesel prices.

VI. REFERENCE/RESOURCE MATERIALS:

Graphing calculators, graph paper, computers (optional)

VII. PRIMARY ASSESSMENT STRATEGIES:

The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric.

Student uses appropriate method to analyze data – 5 points

Student finds correct relationship between the variables included in the table – 5 points

Student correctly uses found relationship to draw conclusions and make predictions – 5 points

Student's work displays all steps and calculations used to determine relationships – 5 points

Student's graphs are clear and easy to interpret – 5 points

Student's work is neat and easy to read – 5 points

VIII. EVALUATION CRITERIA:

Checklist titled Math Processes Checklist will be used to evaluate the mathematical content.

Rubric titled Group Task Rubric will be used to evaluate students' performance as part of a group.

Assessment List will be used to evaluate different aspects of students' work.

IX. INSTRUCTIONAL TIME:

One 90 minute class period

At the Gas Pump

Strand
Algebra and Functions

Mathematical Objective(s)
The goal of the activity is to allow students to use their knowledge of functions to describe a relationship between the variables and make predictions about future prices.

Related SOL
A.11 (analyze data using mathematical models)
AII.9 (analyze data using mathematical models)
AFDA.3 (analyze data using mathematical models)

NCTM Standards
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationship.
- Draw reasonable conclusions about a situation being modeled.
- Approximate and interpret rates of change from graphical and numerical data.

Materials/Resources
- Classroom set of graphing calculators
- Graph Paper
- Computers (optional)

Assumption of Prior Knowledge
- Students should know how to use data to find the best mathematical model.
- Students should be able to interpret an equation based on the situation described in the real-world problem.
- Students should be able to change units according to the situation presented in the real- world problem.
- Students should be able to manipulate between different representations of a function in order to determine certain values of the variable.
- Students are analyzing the impact of crude oil prices on the wholesale price of gas and diesel, and ultimately how they impact our lives.

Introduction: Setting Up the Mathematical Task

The teacher will introduce the task by asking students if they know how prices of gas are calculated. Also discuss recent history of gas and diesel. “How many of you know that gas prices are much higher than they used to be?” “How are gas prices calculated?” “How many of you know what crude oil is?” A discussion around these questions should take place to draw upon students’ prior knowledge.

- Students will work in pairs to complete the activity. Students will develop correlations, and describe them using multiple representations including equations and graphs.
- The teacher will give students the handout that contains the questions and the data.

Student Exploration

The students will use different representations to find relationship between crude oil prices and gasoline and diesel prices. Students may use the calculator, or they may use graph paper to graph the points. If computers are available, the data is available at: http://www.eia.gov/petroleum/data.cfm

Student/Teacher Actions:

- The students will analyze the table and debate on whether there is any correlation between crude oil prices and gasoline/diesel prices.
- The teacher will circulate to ensure students are on the right track. The teacher may help students that don't know where and how to begin, by asking questions such as:" What is correlation?" "What does it mean that two variables are related?" These questions are leading students to think about a function.
- Upon forming an opinion, the students will proceed to investigate whether a relationship exists.
- The students will use the calculator (or graph paper) to plot the data, and find the model that best fits the data.
- The teacher will assist students that are struggling by asking questions such as: "What does a scatter plot look like when there is a correlation?" "What can you conclude after plotting the points?"
- The students will then move on to using the model to predict gasoline and diesel prices for every year until 2040.
- Towards the end of the activity, the students will decide whether a local newspaper's claim is true.
- The teacher will need to circulate to remind students to reach their decision based on facts, and not opinion.
- The students will analyze the data about crude oil production, and investigate its impact on gas and diesel prices.

Whole Class Sharing/Discussion (if relevant)

- At the conclusion of this activity the teacher will hold a class discussion to address any obstacles students had and what they did to overcome them.
- The teacher can ask questions such as: "What did you do in order to determine whether your data is correlated or not?" "What made you think there is any correlation?" "How is the data correlated?" "What type of function fits the data?" "Did you find the local newspaper claim to be true?" "How did you make predictions for the future?" "Did you find any relation between crude oil production and gas/diesel prices?"

Monitoring Student Responses

- The students will have to communicate with their partner to express ideas and make suggestions.
- The pairs will collaborate and work together supportively and respectfully.
- The teacher will be available to monitor progress and assist those students in need of support.
- The teacher also will provide extensive questions for the students that are making steady progress; a question that the teacher can use is: "Investigate whether the production of crude oil affects the price of crude oil".
- The activity will close with a whole class discussion. At this time the students can describe the ideas they used and talk about their conclusions. The students and teacher should also talk about new things the students learned while completing the activity.
- The students will be encouraged to use mathematical terms to describe their thinking.

Assessment List and Benchmarks

- The Assessment List will be used by students and teachers to assess their work.
- The rubric titled Group Task Rubric will be used by teachers to assess collaborative work.

- The checklist titled Math Processes Checklist will be used by teachers to assess the mathematical content of students' work.
- The benchmark will be used to assess exemplary work.

At the Gas Pump

1. Each month the Department of Energy releases information related to wholesale prices of crude oil, gasoline, and diesel. Based on the data below, investigate whether gasoline and diesel prices have any correlation to crude oil prices. If so, describe what type of correlation exists. Support your answer using appropriate mathematics.

Year	Crude oil prices ($/barrel)	Gasoline prices ($/gallon)	Diesel prices($/gallon)
1990	24.53	0.883	0.725
1991	21.54	0.797	0.648
1992	20.58	0.787	0.619
1993	18.43	0.759	0.602
1994	17.2	0.738	0.554
1995	18.43	0.765	0.56
1996	22.12	0.847	0.681
1997	20.61	0.839	0.642
1998	14.42	0.673	0.494
1999	19.34	0.781	0.584
2000	30.38	1.106	0.935
2001	25.98	1.032	0.842
2002	26.18	0.947	0.762
2003	31.08	1.156	0.944
2004	41.51	1.435	1.243
2005	56.64	1.829	1.786
2006	66.05	2.128	2.096
2007	72.34	2.345	2.267
2008	99.67	2.775	3.15
2009	61.95	1.888	1.834
2010	79.48	2.301	2.341
2011	94.88	3.05	3.117
2012	94.05	3.154	3.202
2013	97.98	3.049	3.122

2. At the end of last year, the price of crude oil was $97.98 per barrel. Suppose that for the next 26 years the price will proportionally increase to reach $150 per barrel. According to this information, can you find the yearly prices of gas and diesel throughout 2039?
Describe how you know. Support your answer using appropriate mathematics.

3. The local newspaper claimed that in 50 years gas prices will triple. Do you believe this claim to be true? How do you know?

4. The following table represents data about crude oil production. Analyze the data and decide whether there is a relationship between the production of crude oil and the prices of gasoline and diesel. Describe your findings. Support your answer with appropriate mathematics.

Year	Crude oil production(barrels/day)
1990	7355
1991	7417
1992	7171
1993	6847
1994	6662
1995	6560
1996	6465
1997	6452
1998	6252
1999	5881
2000	5822
2001	5801
2002	5744
2003	5649
2004	5441
2005	5181
2006	5088
2007	5077
2008	5000
2009	5350
2010	5482
2011	5645
2012	6497
2013	7441

Assessment List

Name:

	0	2.5	5	Score
Student uses appropriate method to analyze data	The student did not know what to do	The student analyzed the data but did not use an appropriate method	The student used a method that was appropriate	___/5
Student finds correct relationship between the variables included in the table	The student did not find a relationship between the variables included in the table	The student found a relationship between variables, but it was incorrect	The student found correct relationship between the variables in the table	___/5
Student correctly uses found relationship (equation) to draw conclusions and make predictions	The student did not draw conclusions and make predictions	The student used the relationship but made errors in calculations	The student used the relationship correctly and his/her conclusions and predictions are correct	___/5
Student's work displays all steps and calculations used to determine relationships	The student showed inconclusive work and calculations	The student showed work and calculations only partially	The student showed all his/her work and calculations	___/5
Student's graphs are clear and easy to interpret	The student did not label the axes of the graph, did not use correct units to represent the variables, and/or the curve is incorrect	The student made a graph, but omitted 1 or 2 of the following: labels on axes, correct units for each variable, correct sketch of curve.	The student has a clear graph that is easy to interpret	___/5
Student's work is neat and easy to read	The student displays work that is not organized or neat	The student displays work that is partially easy to read and neat	The student display work that is neat and easy to read	___/5

Title: Group Task Rubric—Middle and High School

Type of Assessment: Collaboration/Teamwork

4	3	2	1
Understanding of Task			
I/we demonstrated an in-depth understanding of the content, processes, and demands of the task.	I/we demonstrated substantial understanding of the content and task, even though some supporting ideas or details may be overlooked or misunderstood.	I/we demonstrated gaps in our understanding of the content and task.	I/we demonstrated minimal understanding of the content.
Completion of Task			
I/we fully achieved the purpose of the task, including thoughtful, insightful interpretations and conjectures.	I/we accomplished the task.	I/we completed most of the assignment.	I/we attempted to accomplish the task, but with little or no success.
Communication of Findings			
I/we communicated our ideas and findings effectively, raised interesting and provocative questions, and went beyond what was expected.	I/we communicated our findings effectively.	I/we communicated our ideas and findings.	I/we did not finish the investigation and/or were not able to communicate our ideas very well.
Group Process			
We used all of our time productively. Everyone was involved and contributed to the group process and product.	We worked well together most of the time. We usually listened to each other and used each other's ideas.	We worked together some of the time. Not everyone contributed equal efforts to the task.	We really did not pull together or work very productively as a group. Not everyone contributed to the group effort.

4	3	2	1
Problem Solving			
Problems did not deter us. We were proactive and worked together to solve problems.	We worked together to overcome problems we encountered.	We might have worked more productively as a group.	Some people did more work than others. OR Nobody worked very well in the group.

Title: Math Processes Checklist

Student name:

	Consistently, Usually, Rarely/Never	**Dates Observed**	**Notes**
Student determines the problem.			
Student asks appropriate questions.			
Student chooses appropriate strategies.			
Student checks answers.			
Student justifies answers.			
Student sees relationships among concepts.			
Student applies strategies effectively.			
Student uses more than one strategy to solve a problem.			
Student uses mental math.			
Student plans and revises.			
Student shows work.			
Student generalizes processes to other situations.			
Student demonstrates task commitment.			

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Sample of Exemplary work

1. Student changes the units correspondingly:

1 barrel of crude oil = 42 gallons

Year	Crude oil prices ($/barrel)	Gasoline prices ($/gallon)	Diesel prices($/gallon)
1990	24.53 = 0.58 $/gallon	0.88	0.73
1991	21.54 = 0.51 $/gallon	0.80	0.65
1992	20.58 = 0.49 $/gallon	0.79	0.62
1993	18.43 = 0.44 $/gallon	0.76	0.60
1994	17.2 = 0.41 $/gallon	0.73	0.56
1995	18.43 = 0.44 $/gallon	0.77	0.56
1996	22.12 = 0.53 $/gallon	0.85	0.68
1997	20.61 = 0.49 $/gallon	0.84	0.64
1998	14.42 = 0.34 $/gallon	0.67	0.49
1999	19.34 = 0.46 $/gallon	0.78	0.58
2000	30.38 = 0.72 $/gallon	1.11	0.94
2001	25.98 = 0.62 $/gallon	1.03	0.84
2002	26.18 = 0.62 $/gallon	0.95	0.76
2003	31.08 = 0.74 $/gallon	1.16	0.94
2004	41.51 = 0.99 $/gallon	1.44	1.24
2005	56.64 = 1.35 $/gallon	1.83	1.79
2006	66.05 = 1.57 $/gallon	2.12	2.10
2007	72.34 = 1.72 $/gallon	2.35	2.27
2008	99.67 = 2.37 $/gallon	2.78	3.15
2009	61.95 = 1.48 $/gallon	1.89	1.83
2010	79.48 = 1.89 $/gallon	2.30	2.34
2011	94.88 = 2.26 $/gallon	3.05	3.12
2012	94.05 = 2.24 $/gallon	3.15	3.20
2013	97.98 = 2.33 $/gallon	3.05	3.12

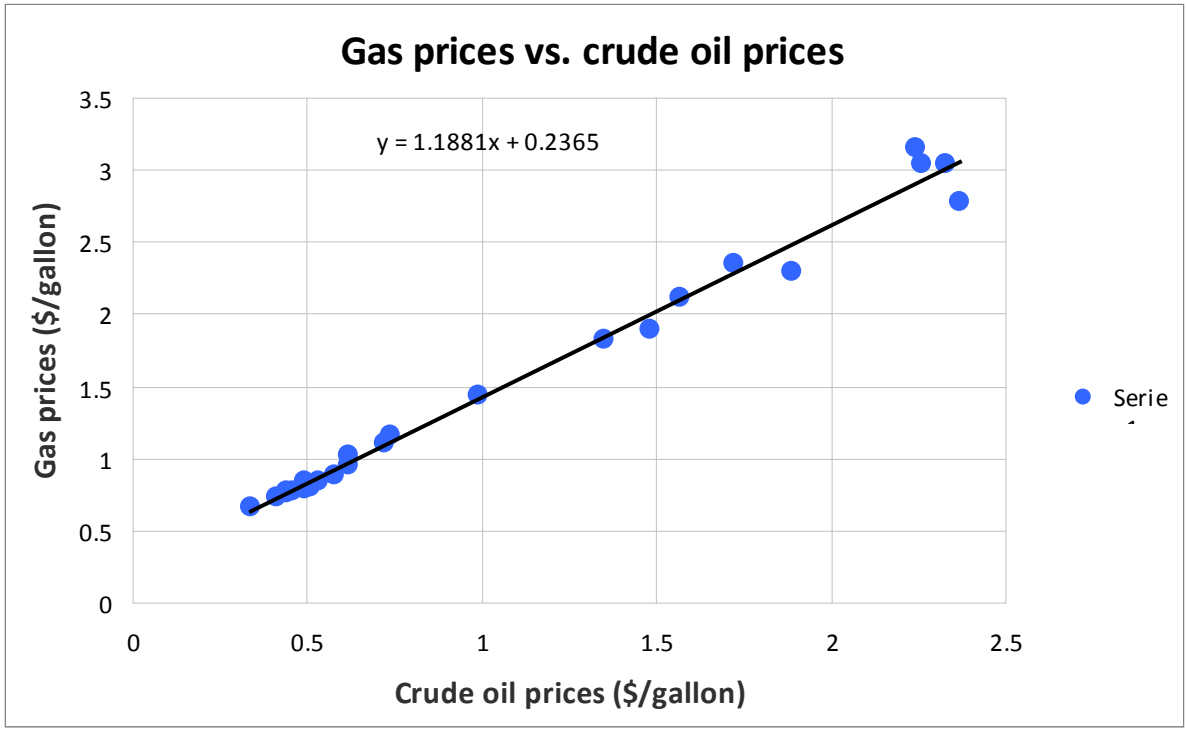
After graphing the scatter plot of gasoline prices vs. crude oil prices, I concluded that there is a correlation between crude oil prices and gasoline prices; the model that best fits the data is linear. The equation of the function is:

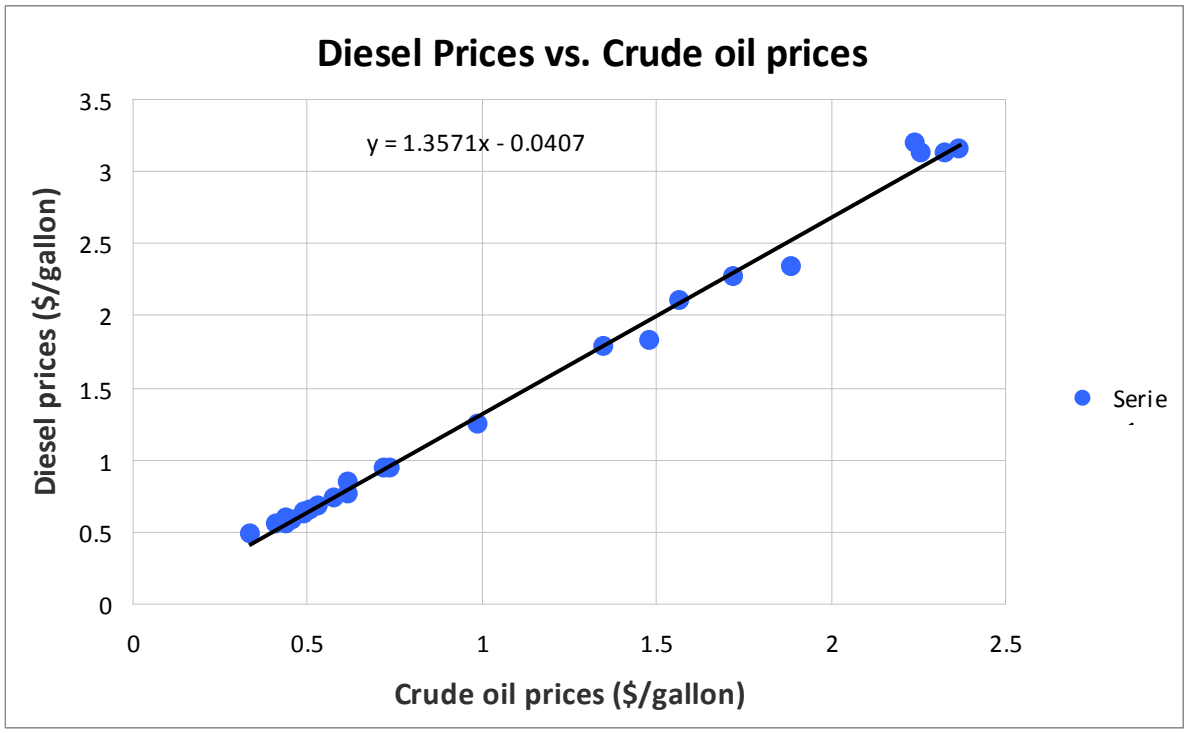
$y = 1.188x + 0.237$, where x represents the price of crude oil and y represents the price of gasoline. I found this equation using the STAT feature of the calculator.

After graphing the scatter plot of diesel prices vs. crude oil prices, I concluded that there is a correlation between crude oil prices and diesel prices; the model that best fits the data is linear. The equation of the function is:

$y = 1.357x - 0.041$, where x represents the price or crude oil and y represents the price of diesel.

NOTE: the prices from the original table have been rounded to 2 digits to reflect everyday $ amounts.





2. Because the increase of crude oil prices over the next 26 years is linear, we can find the prices of crude oil by doing the following:

$150 - 97.98 = 52.02$

$52.02 \div 26 = 2.000769$

Each year the price of crude oil increases by approximately $2 per year. Using the linear functions found in #1, we can find the prices of gasoline and diesel by replacing the values of *x* with the corresponding numbers in the **Crude oil prices** column. I found that the easiest way to do this was to insert the linear equations into "***y*=**", then use the "**Ask**" feature of the "**Table**" to type the values of *x*.

Year	**Crude oil prices ($/barrel)**	**Gasoline prices ($/gallon)**	**Diesel prices ($/gallon)**
2014	99.98 = 2.38 $/gallon	3.06	3.19
2015	101.98 = 2.42 $/gallon	3.11	3.24
2016	103.98 = 2.48 $/gallon	3.18	3.32
2017	105.98 = 2.52 $/gallon	3.23	3.38
2018	107.98 = 2.57 $/gallon	3.29	3.45
2019	109.98 = 2.62 $/gallon	3.35	3.51
2020	111.98 = 2.67 $/gallon	3.41	3.58
2021	113.98 = 2.71 $/gallon	3.46	3.64
2022	115.98 = 2.76 $/gallon	3.52	3.70
2023	117.98 = 2.81 $/gallon	3.58	3.77
2024	119.98 = 2.86 $/gallon	3.63	3.84
2025	121.98 = 2.90 $/gallon	3.68	3.89
2026	123.98 = 2.95 $/gallon	3.74	3.96
2027	125.98 = 3.00 $/gallon	3.80	4.03
2028	127.98 = 3.05 $/gallon	3.86	4.10
2029	129.98 = 3.09 $/gallon	3.91	4.15
2030	131.98 = 3.14 $/gallon	3.97	4.22
2031	133.98 = 3.19 $/gallon	4.03	4.29
2032	135.98 = 3.24 $/gallon	4.09	4.36

2033	137.98 = 3.29 $/gallon	4.15	4.42
2034	139.98 = 3.33 $/gallon	4.19	4.48
2035	141.98 = 3.38 $/gallon	4.25	4.55
2036	143.98 = 3.43 $/gallon	4.31	4.61
2037	145.98 = 3.48 $/gallon	4.37	4.68
2038	147.98 = 3.52 $/gallon	4.42	4.74
2039	149.98 = 3.57 $/gallon	4.48	4.80

3. 50 years from now the year will be 2064. Since we already know the price of crude oil in 2039, we can use this information to predict the price of crude oil for 2064. There is a difference of 25 years between 2039 and 2064, and since the price of crude oil is assumed to increase by approximately $2 a year, we can conclude that in 2064 the price of crude oil will be:

$$149.98 + 2 \cdot 25 = 199.98$$

So in 2064 the price of crude oil is predicted to be $199.98/barrel

$199.98 \div 42 = 4.76$, so the price of crude oil is predicted to be $4.76/gallon.

Now we can use the equation of the line of best fit to predict the price of gasoline:

$y = 1.188 \cdot 4.76 + 0.237 = 5.8919$

So the predicted price of gasoline is $5.89/gallon.

If we compare that to today's wholesale price:

$5.89 \div 3.06 = 1.92$

We can conclude that the newspaper's claim was incorrect. 50 years from now the price of gas will be close to double, and not triple that of today.

4. Note: answers may vary.

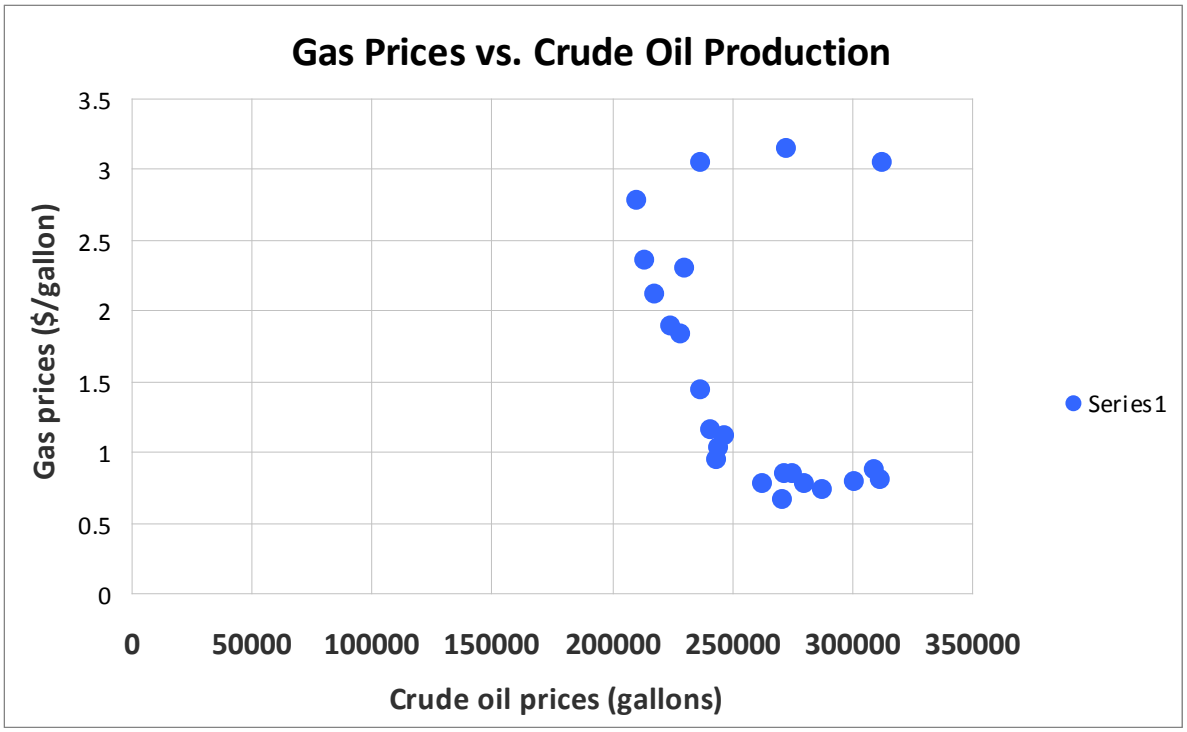
The table below represents yearly crude oil production in barrels and gallons, wholesale gasoline prices, and wholesale diesel prices.

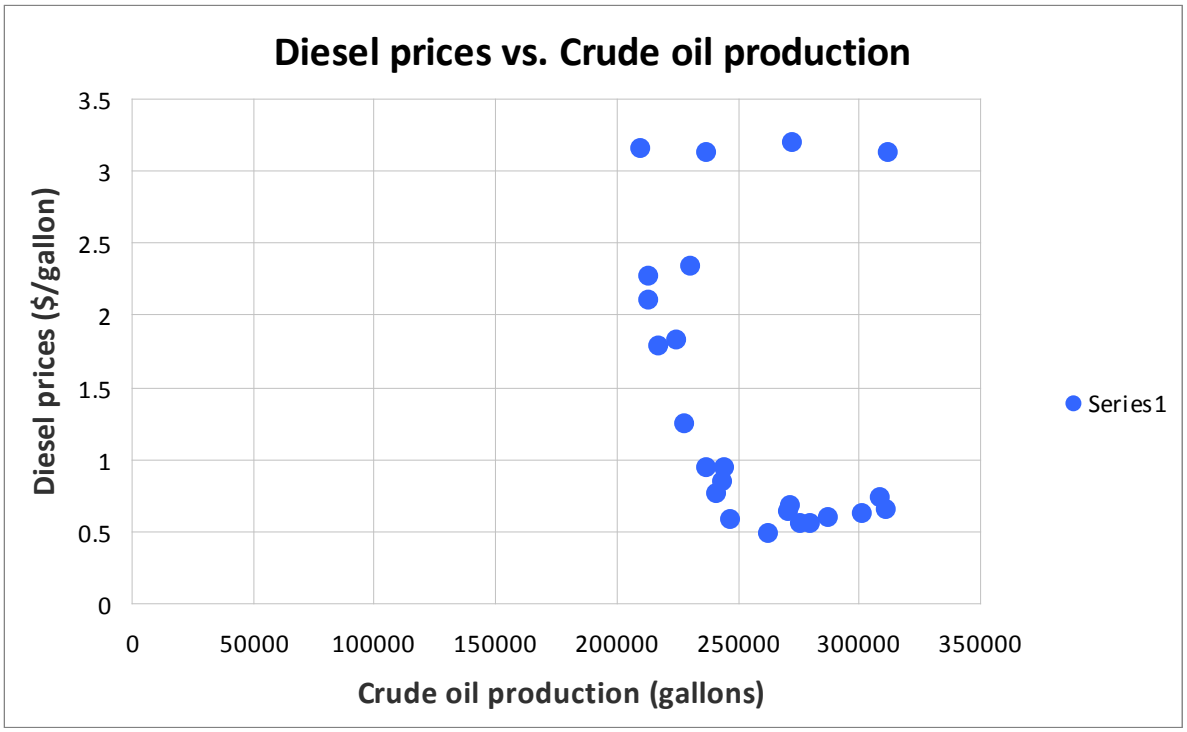
Year	**Crude oil production(barrels/day)**	**Gasoline prices ($/gallon)**	**Diesel prices($/gallon)**
1990	7355 = 308910 gallons	0.88	0.73
1991	7417= 311514 gallons	0.80	0.65

1992	7171 = 301182 gallons	0.79	0.62
1993	6847 = 287574 gallons	0.76	0.60
1994	6662 = 279804 gallons	0.73	0.56
1995	6560 = 275520 gallons	0.77	0.56
1996	6465 = 271530 gallons	0.85	0.68
1997	6452 = 270984 gallons	0.84	0.64
1998	6252 = 262584 gallons	0.67	0.49
1999	5881 = 247002 gallons	0.78	0.58
2000	5822 = 244524 gallons	1.11	0.94
2001	5801 = 243642 gallons	1.03	0.84
2002	5744 = 241248 gallons	0.95	0.76
2003	5649 = 237258 gallons	1.16	0.94
2004	5441 = 228522 gallons	1.44	1.24
2005	5181 = 217602 gallons	1.83	1.79
2006	5088 = 213696 gallons	2.12	2.10
2007	5077 = 213234 gallons	2.35	2.27
2008	5000 = 210000 gallons	2.78	3.15
2009	5350 = 224700 gallons	1.89	1.83
2010	5482 = 230244 gallons	2.30	2.34
2011	5645 = 237090 gallons	3.05	3.12
2012	6497 = 272874 gallons	3.15	3.20
2013	7441 = 312522 gallons	3.05	3.12

After plotting the scatter plot of gasoline prices versus crude oil production, I concluded that there is no obvious correlation between the two variables. The only model that comes close to the data is quadratic, but even so, the graph of that function does not come close to the data points. It also appears that up until the year 2000, as the production decreases, the prices of gas are decreasing as well. Then the production is continuing to decrease, but prices are rapidly going up. Afterwards, beginning with 2009 the production of crude oil is increasing, and prices are increasing as well. The models that I investigated upon plotting the points are the following: exponential, quadratic, and linear. A similar situation occurred when investigating the diesel prices versus crude oil production. There are potentially many more variables that affect the prices of gasoline and diesel, not just crude oil production.

The graphs below show how there is no correlation between crude oil production and gas/diesel prices.


Gas Prices vs. Crude Oil Production
Gas prices ($/gallon)
3.5
3
2.5
2
1.5
1
0.5
0
0
50000
100000
150000
200000
250000
300000
350000
Crude oil prices (gallons)
Series1


Diesel prices vs. Crude oil production
Diesel prices ($/gallon)
3.5
3
2.5
2
1.5
1
0.5
0
0
50000
100000
150000
200000
250000
300000
350000
Crude oil production (gallons)
Series1